A Novel Method to Investigate Capillary Waves on Liquid Surfaces

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Introduction

Aim

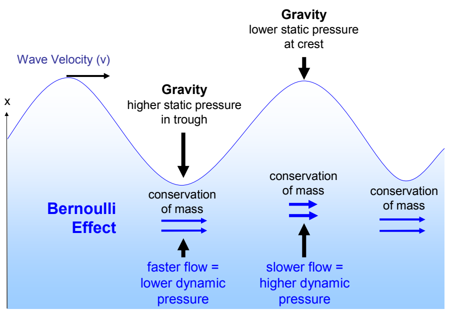
- Develop a new experiment for the second year laboratory.
- Demonstrate the difference between capillary & gravity waves.
- Create a fun, stimulating experiment.
- Experimental setup should be simple, robust and reliable.

Background

- Experimental idea from a Russian paper [1].
- Measurement of frequency (f) and wavelength (λ) leads to a value for the surface tension (σ) and the acceleration due to gravity (g) (see figure 4).

Theory

Einstein's Gravity Wave Model



- Einstein combined spring oscillation and the Bernoulli effect.
- Einstein's gravity model also applied to surface tension.
- $m\ddot{x} = -kx$ Simple Harmonic Motion equation.
- Gravity waves have gravity as restoring force.
- Capillary waves have surface tension as restoring force.
- Equating opposing pressures produces velocity equation (1). [2][3][4]

$$v = \sqrt{\frac{\lambda g}{2\pi} + \frac{2\pi\sigma}{\rho\lambda}} \qquad (1)$$

Gravity Term — Surface Tension Term

g - Acceleration due to Gravity

σ - Surface Tension

Theory (Continued)

Theoretical Graph of Wave Velocity as a Function of Wavelength Using Equation (1)

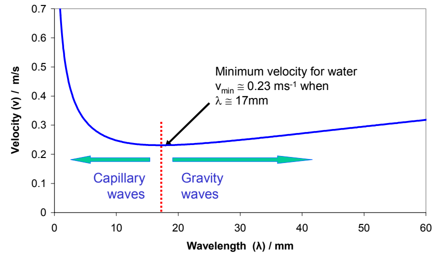


Figure 1. Theoretical plot of velocity as function of wavelength using equation (1). The surface tension (σ) has been taken as 72×10^{-3} Nm, acceleration due to gravity (g) as 9.81 ms^{-2} and the density of water (ρ) as 1000 kgm^{-3}.

- Gravity waves big waves, λ >17mm (for water).
- Capillary waves small, λ < 17mm (for water).
- Waves around minimum velocity influenced by both gravity and surface tension

Figure 2. Capillary Waves occurring in nature on a beach

Method

Experimental Setup

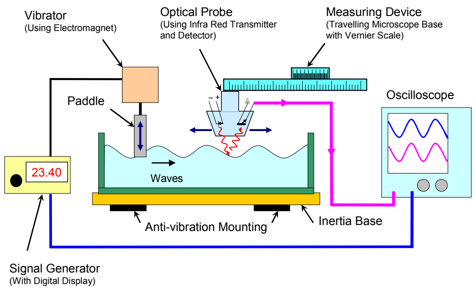


Figure 3. A schematic diagram of the experimental setup.

- Signal generator powers the electromagnetic vibrator and governs the frequency.
- Vibrator drives paddle which creates small amplitude plane waves, propagating at the vibrator frequency.
- Probe moved horizontally over liquid surface, wavelengths counted as probe trace (pink) moves relative to static signal trace (blue) on oscilloscope.
- Distance moved divided by number of wavelengths traversed gives mean wavelength.

Results

Linear Plot of Experimental Results

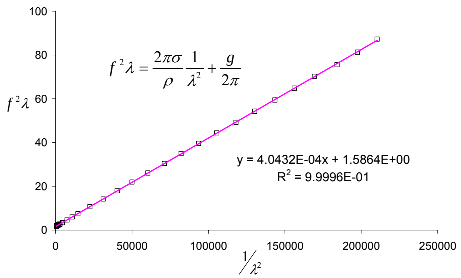


Figure 4. Equation (1) rearranged to produce a linear plot. From the plot, surface tension (σ) can be found from the gradient (64.35±1.6 mNm^{-1}). Acceleration due to gravity (g) can be found from the intercept with the y-axis (9.967±0.2ms^{-2}). The Error bars are within the black boxes.

Comparing Experimental Results to Theory for Wave Velocity as Function of Wavelength

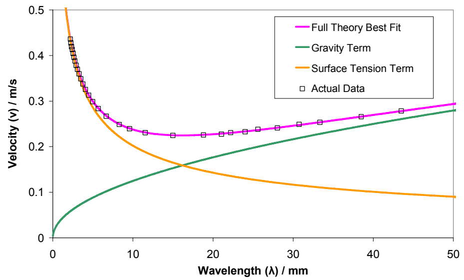


Figure 5. Velocity as a function of wavelength with the surface tension (σ) taken as 64.35mNm^{-1}. The error bars are within the black boxes. The gravity term takes the surface tension (σ) as zero and the surface tension term takes gravity (g) as zero.

Comparing Experimental Results to Theory for Wave Frequency as Function of Wavelength

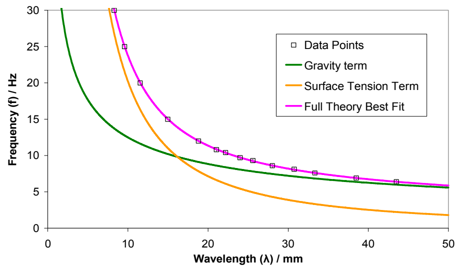
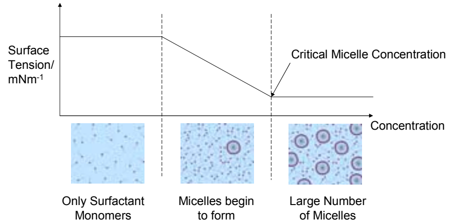


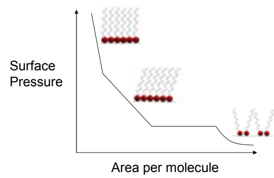
Figure 6. Frequency as a function of wavelength with the surface tension (σ) taken as 64.35mNm^{-1}. The error bars are within the black boxes. The gravity term takes the surface tension as zero and the surface tension term takes gravity as zero.

Further Study

- The apparatus could be used to investigate surfactants
  - For soluble surfactants one can find the **C**ritical **M**icelle **C**oncentration



- For insoluble surfactants one can measure the surface pressure – area isotherm.
- Analogous to a phase changes in a pressure volume diagram.



Conclusion / Discussion

- Results correlate very well with theoretical model.
- The experiment meets the criteria for the 2nd year laboratory.
- Problems with an electrochemical technique were overcome using an optical detector.
- Optical probe is a non-invasive simple experimental technique.
  - Allows measurement of waves at very low amplitudes.
  - Low amplitudes (< λ /2) needed, as assumed in model.
  - Low amplitudes decay before being reflected.
- Optical detector simple, economical and reliable.
- Many possible extensions using surfactants

References

[1] I.I.Popva, P.S.Bulkin (1998): *Measurement of Surface Tension by Means of Excited Capillary Waves.* General Physics Laboratory - Molecular Physics, Moscow University Press

[2] Kenyon, Kern E, (1998): *Capillary Waves Understood by an Elementary Method*. Journal of Oceanography, Vol.54 pp 343 to 346

[3] Landau LM and Lifshitz EM (1959): *Fluid Mechanics*. Pergamon Press Ltd., (translated by Sykes & Reid)

[4] Einstein, A, (1916): *Elementare Theorie der Wasserwellen und des Fluges*. Naturwissenschaften, 4, 509, (translated Kenyon & Shears).

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